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# The LASSO – A Novel Method for Predictive Covariate Model Building in Nonlinear Mixed Effects Models

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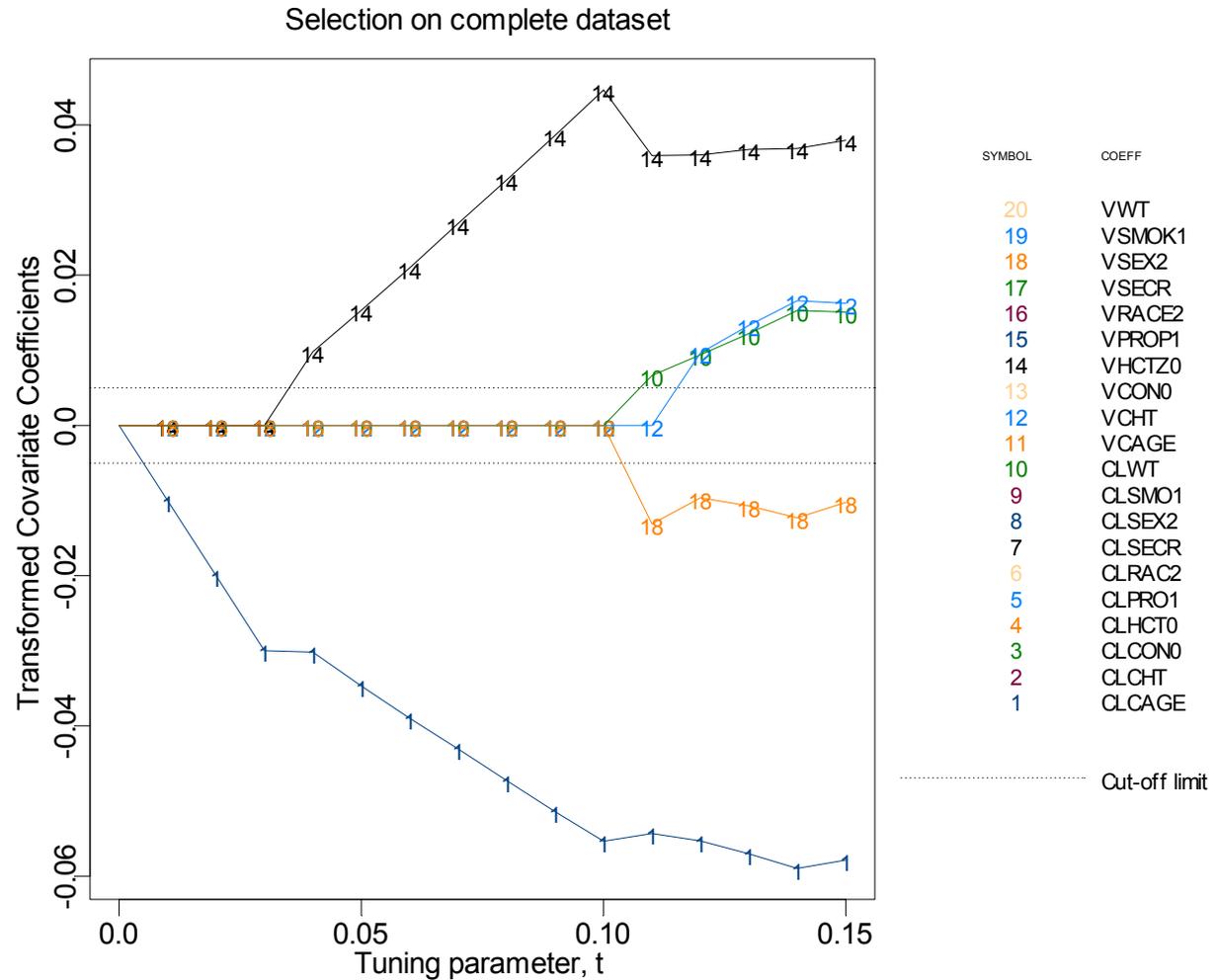
# Introduction - Covariate Selection in Nonlinear Mixed Effects Models

- Stepwise-Covariate Modelling (SCM)
- Some of the problems with SCM:
  1. Border-line significant covariate effects either discarded or included
    - The LASSO would shrink these covariate effects but may keep them in the model
  2. User must specify p-value for selection
    - The LASSO uses cross-validation
  3. Long computer-run-times
    - The LASSO may be faster

# Theory - LASSO for “Least Absolute Shrinkage and Selection Operator”

- Covariate transformation
  - centred around zero
  - normalised to between-individual standard deviation (unless time-varying covariates)
- Covariate-coefficient magnitude on same scale
- Estimating the lasso model:
  - estimating full covariate model with restriction
    - absolute sum of covariate coefficients  $\leq t$
  - $t$  (tuning parameter), determines Model Size

# Theory - Illustration of the LASSO-Estimates over t





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# Objectives

To implement the LASSO for covariate selection within NONMEM and to compare this method to the commonly-used SCM

# Method – Implementation of the LASSO

- Implemented as a fully automated tool using Perl-speaks-NONMEM (PsN)
- Optimal  $t$  estimated using cross-validation
  - Cross validation similar to data splitting but uses data more efficiently
  - Five-fold cross validation on NONMEM objective function value (OFV)

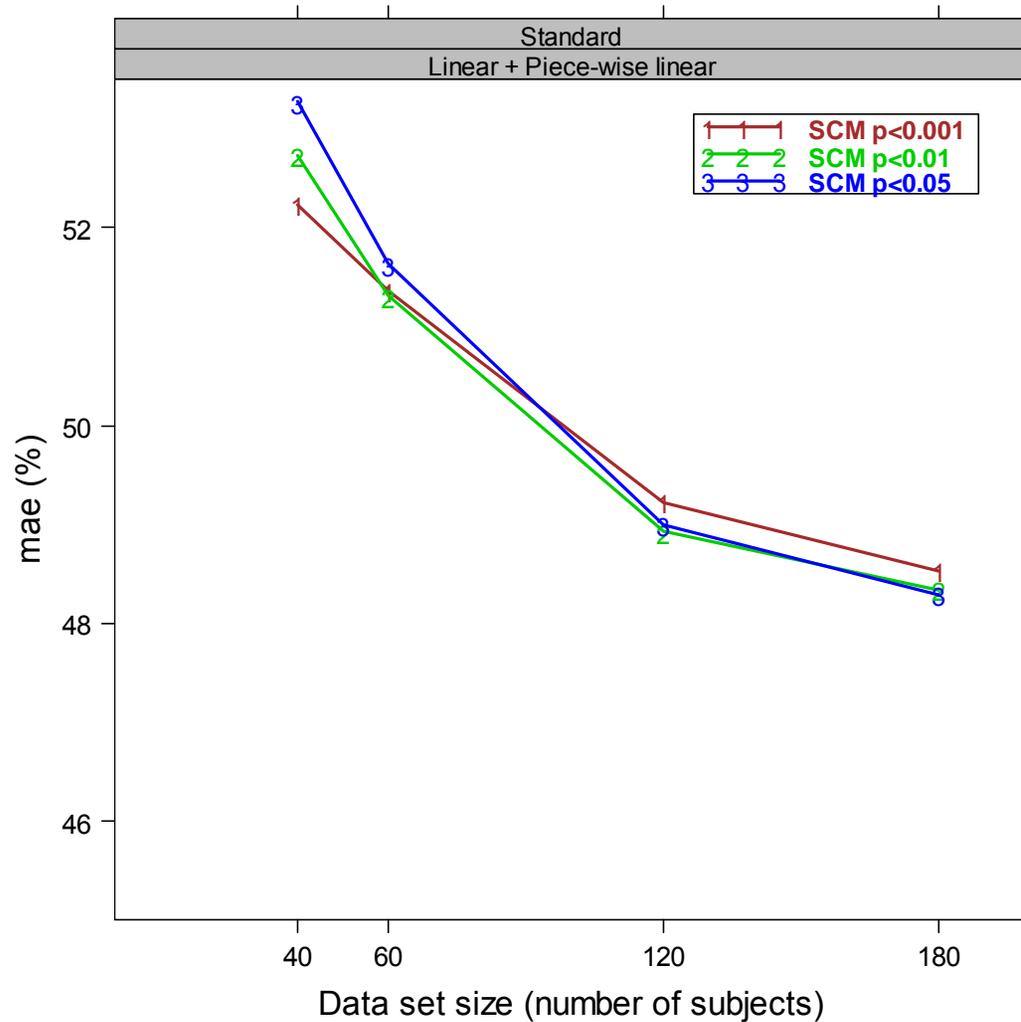
# Method – Creation of Analysis Datasets

- Analysis datasets generated by sampling subjects (with replacement) from a PK dataset containing 721 subjects
  - 40, 60, 120 or 180 subjects in each analysis dataset
  - 100 replicate dataset of each size

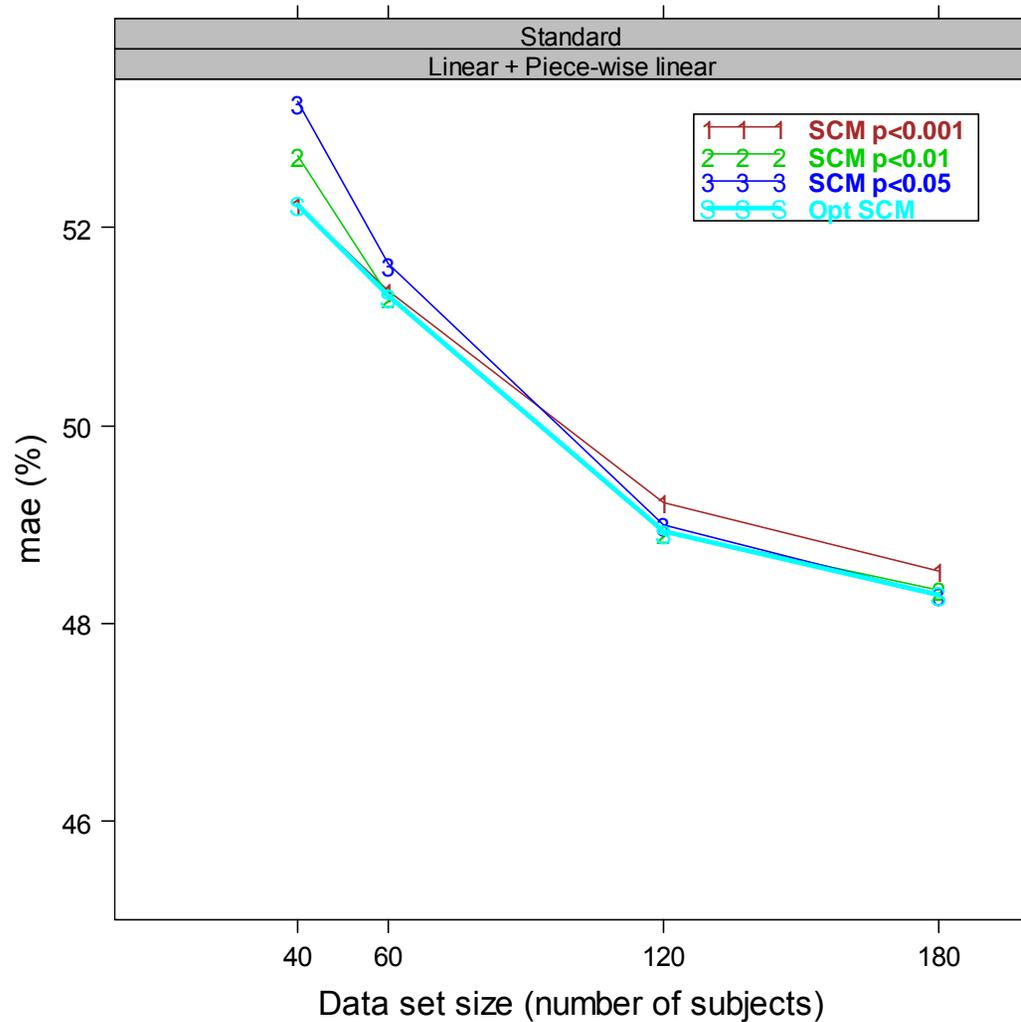
# Method – Creation of Validation Datasets

- For each analysis dataset a validation dataset was created comprising all subjects among the 721 that were not in the corresponding analysis dataset
- To compare models produced by SCM and LASSO, prediction error evaluated on observations in validation dataset:
  - $\text{mae} = \text{average}(|\text{obs}_n - \text{pred}_n| / \text{obs}_n) \cdot 100\%$

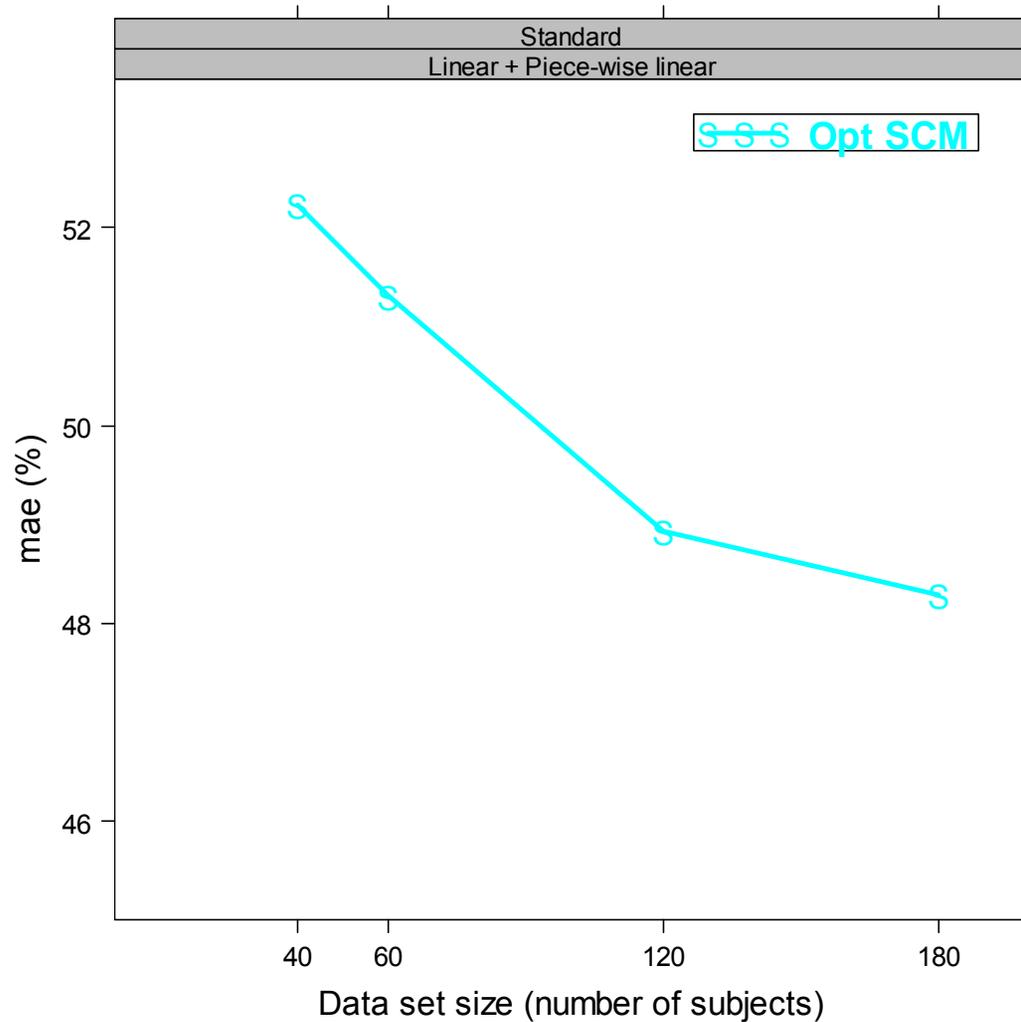
# Results – Prediction Error for the SCM with Different p-values



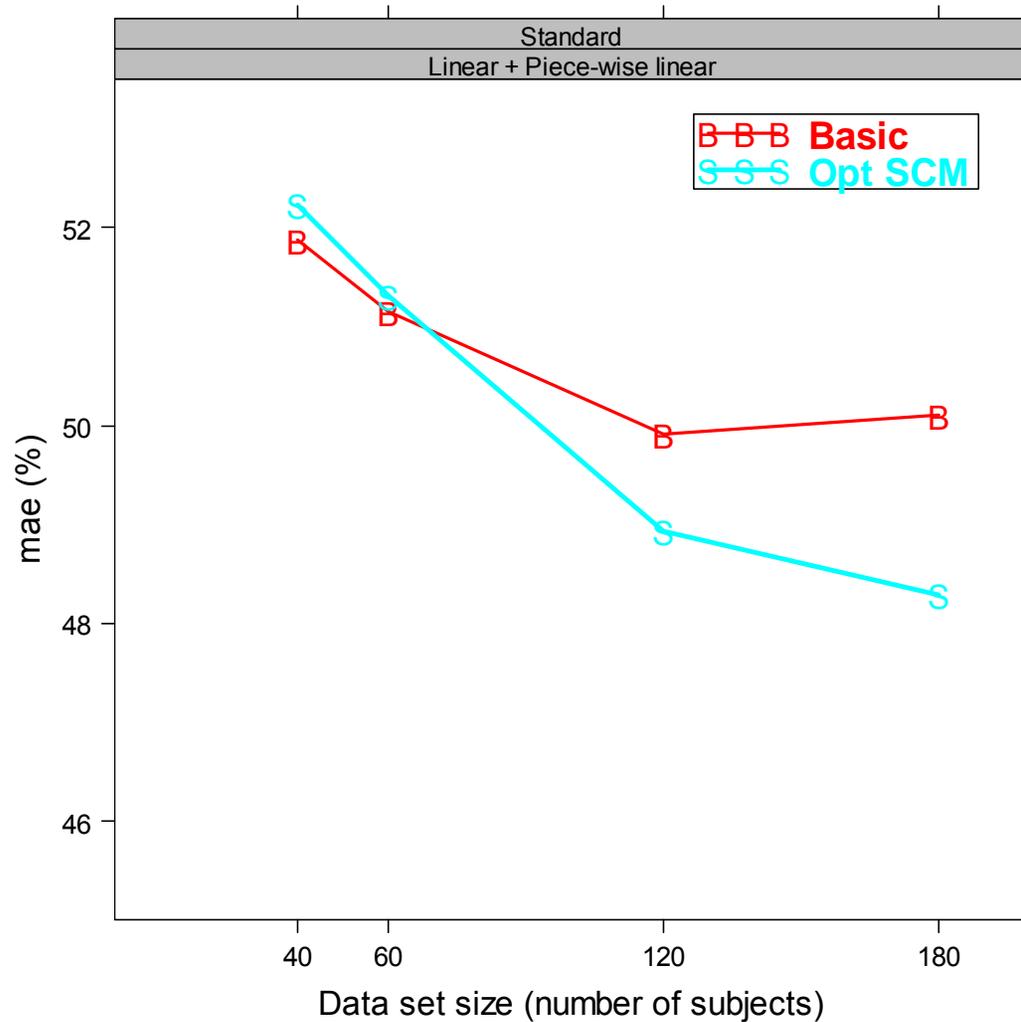
# Results – Prediction Error for the SCM with Different P-Values



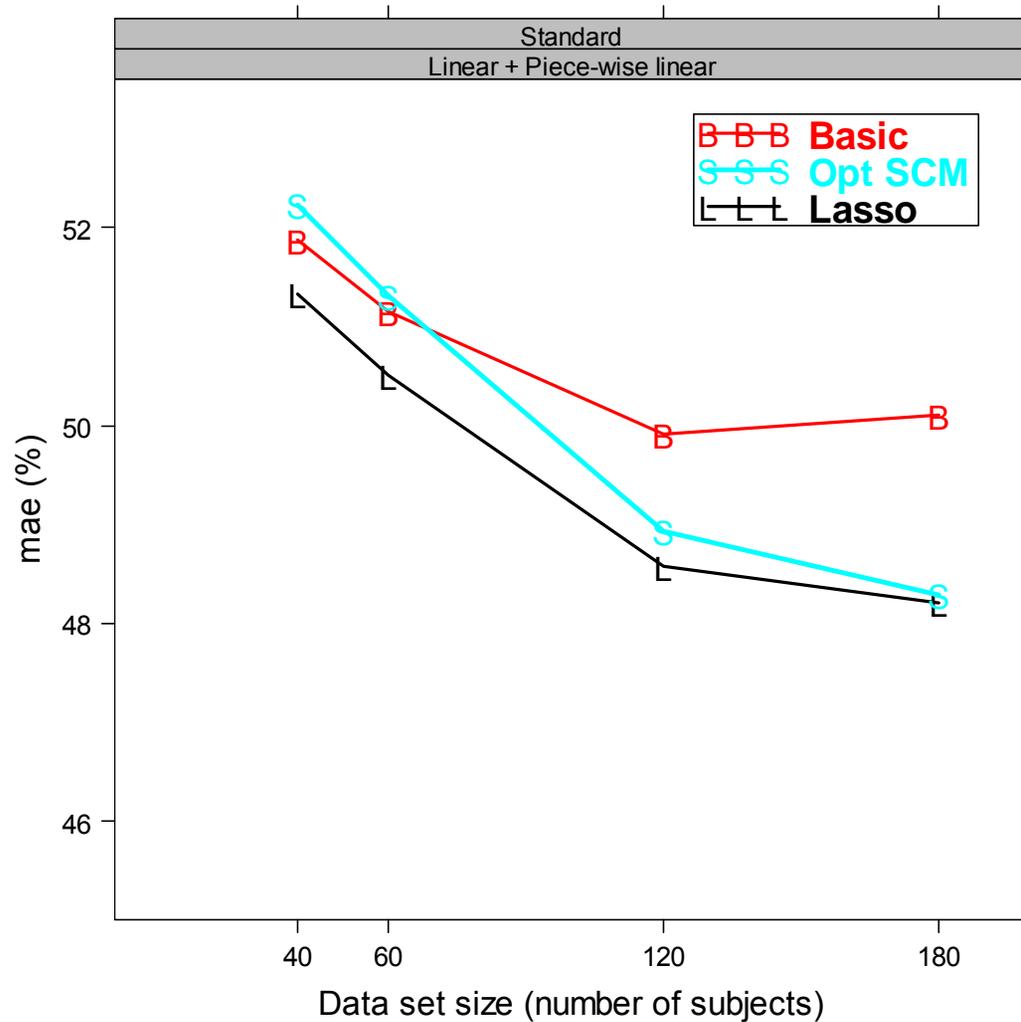
# Results – Prediction Error for SCM



# Results – Prediction Error for SCM and Starting Model

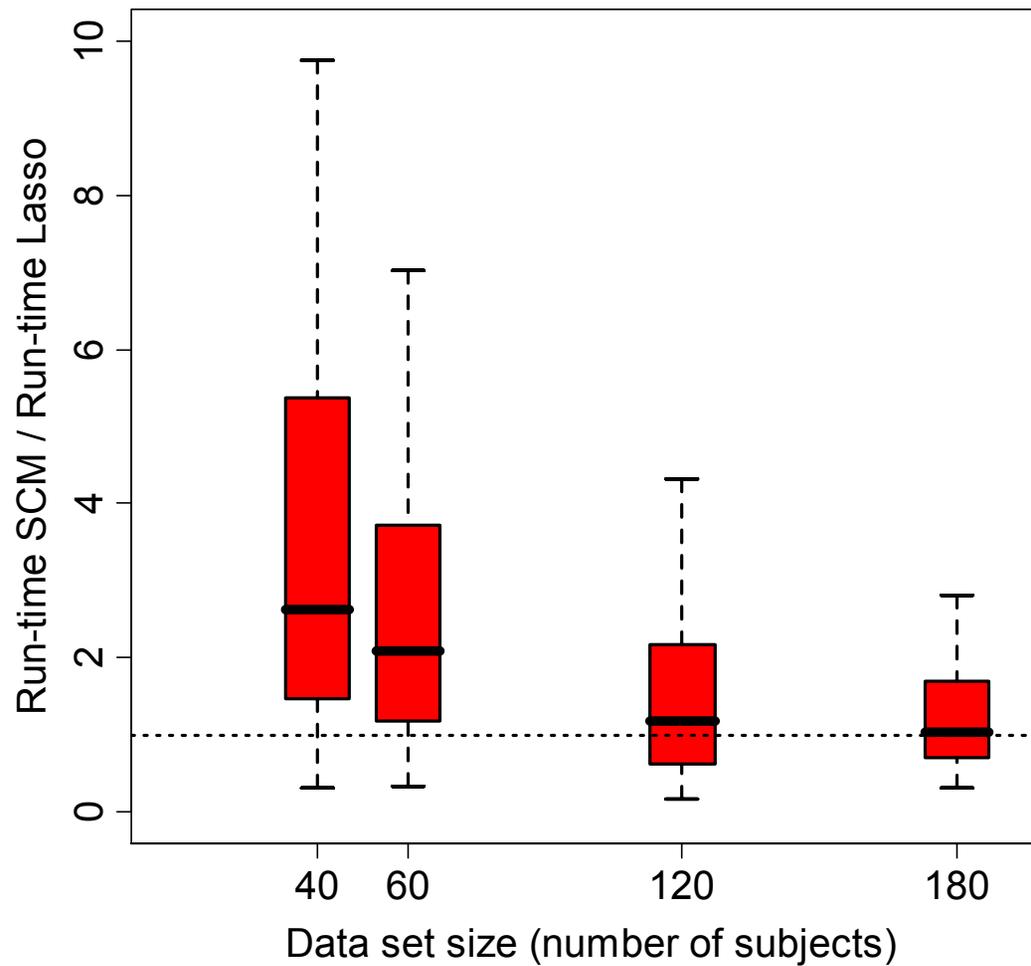


# Results – Prediction Error for SCM, LASSO & Starting Model

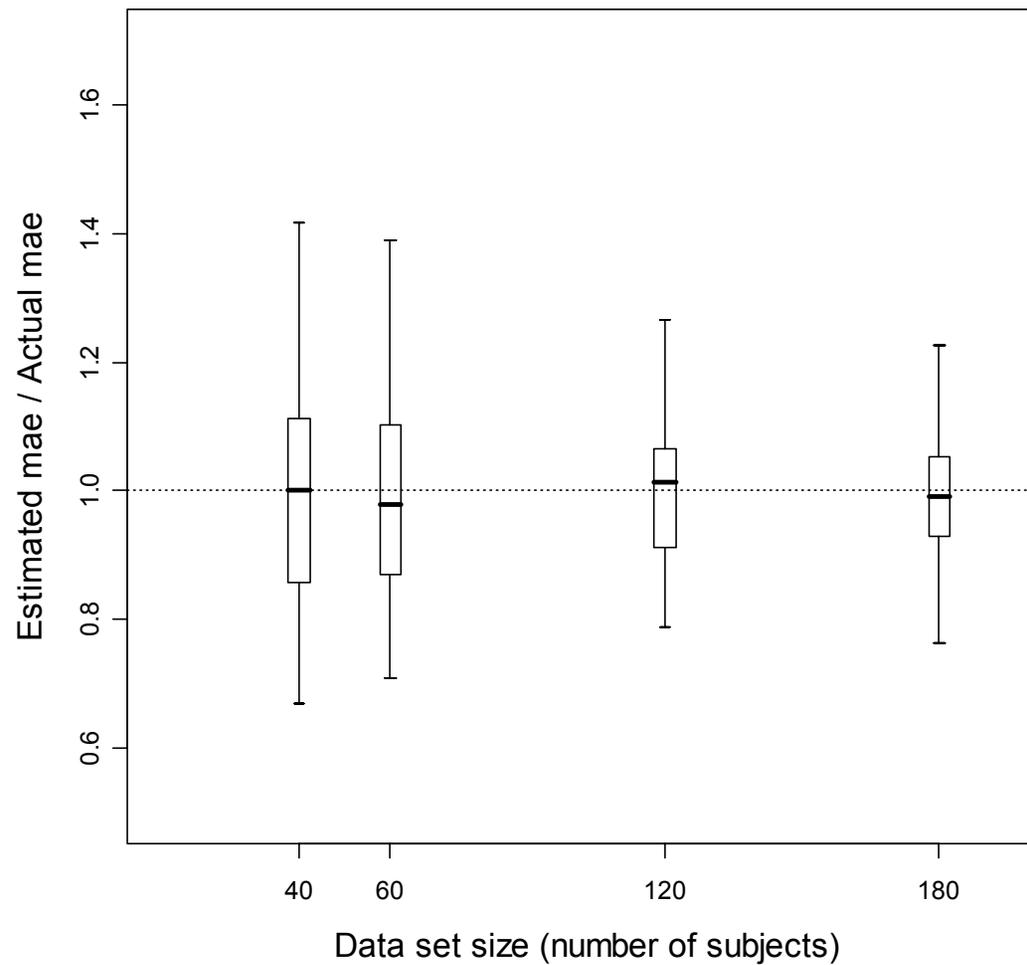




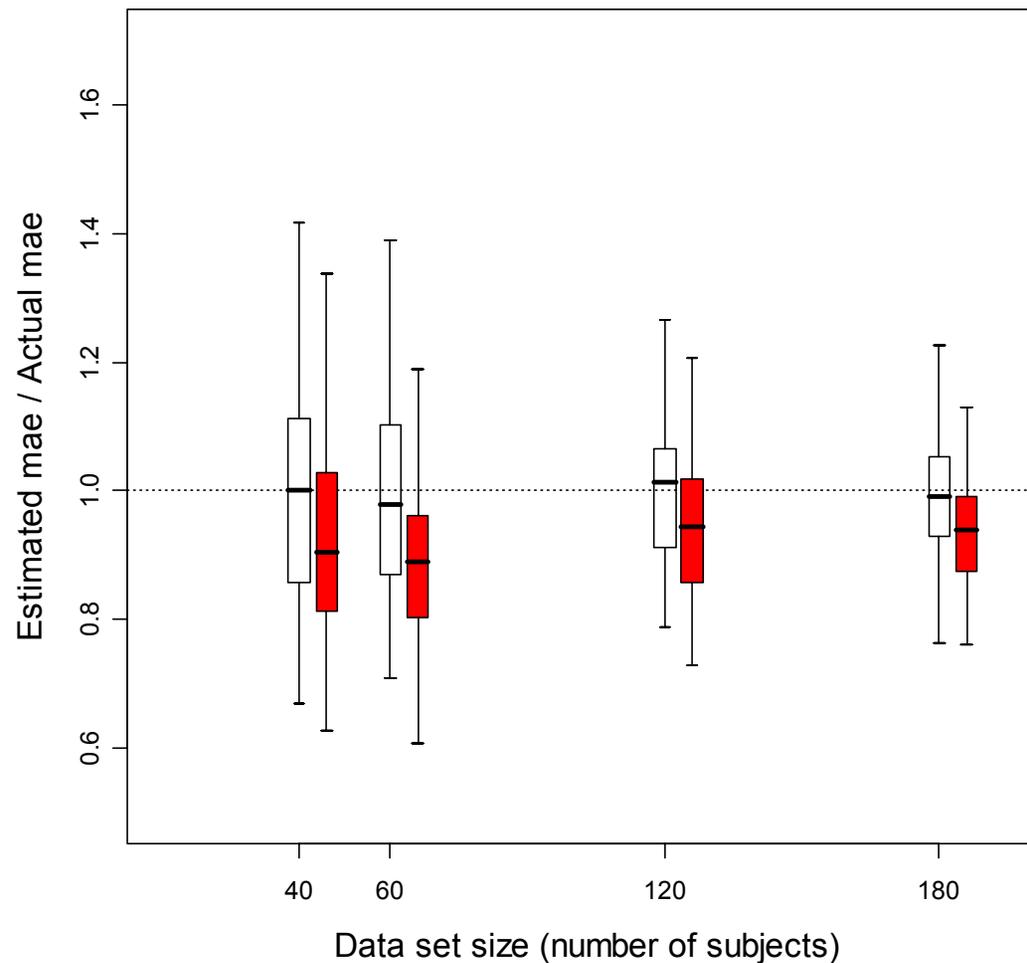
# Results – Computer Run-Time



# Results – LASSO Provides Unbiased Estimate of Prediction Error



# Results – SCM Provides No Accurate Estimate of Prediction Error



# Discussion and Conclusions – Drawbacks of the LASSO

- May produce a more complex model
- Cross-validation difficult on unstable model
  - Estimable on 80% of the original data
- Little experience of this method in pop PK/PD

# Discussion and Conclusions – Advantages of the LASSO

- The LASSO is preferable for small datasets
- Better predictive performance
    - Also for small subpopulations in large datasets!
  - Shorter run-time if many covariate relations
- 
- No need to specify a p-value for selection
  - Provides estimate of prediction error
    - Covariate-model selection taken into account
    - External validation of covariate model!

# Take-Home Message

On small datasets use the  
LASSO rather than the SCM



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